The IRiMaS Software: Integrating interactive listening and play into musicological research

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ABSTRACT

This paper introduces the IRiMaS software, developed within a European Research Council funded project based at the University of Huddersfield (2017-2022). The IRiMaS project (Interactive Research in Music as Sound) aims at developing new modes of musicology, by placing the aural, listening, and play at the forefront of the research into musical form, processes, and practices. The primarily considered repertoires are those in which traditional notation is fundamentally problematic for the analysis, such as ethnomusicology, improvised music, or branches of contemporary music like spectral music in which the gap between what is written on the score and what is heard is often considerable. For such musical domains, an interactive aural approach, relying on software and sound analysis, is potentially a useful alternative to notation, as it can lead to the scrutiny and analysis of music on the primary basis of listening. In this context, the IRiMaS software is a platform through which the musicologist can build her/his entire workflow, from the simple investigation of a sound file to the elaboration of advanced interactive analytical charts, by providing an important library of tools to support all the dimensions of the research itself and of the dissemination of its outcomes.

1. Introduction

While it is arguably intuitive to consider that the study of music should have sound as its primary medium and listening as the basis for its methodology, the importance of classical notation in the development of music theory, music analysis, and musicology as academic disciplines has led to a strong focus on the written score for research into many repertoires and musical practices, including those which do not rely on notation as much as classical western music. Yet, listening and emphasizing various sonic aspects of the music enables musicologists and their audiences to understand better the structure of the music, the nuances in timbre, pitch and rhythm sequences; such an approach is particularly important when the music cannot be satisfactorily represented with the traditional score.

The IRiMaS project (Interactive Research in Music as Sound), directed by Michael Clarke at the University of Huddersfield and funded by the European Research Council for five years (2017-2022)[1], aims at developing new modes of conducting and disseminating musicology by giving to the sound and listening the primary role in the research into musical form, processes, and practices.

Such a focus on the aural, however, requires a considerable investment into software development. Certain recent initiatives have attempted to support analytical texts with sound examples via hypermedia links [2], but the philosophy of IRiMaS is to go way beyond simple playback, and to fully integrate a smooth interaction with any aspect of the music that can be perceived as sound to stimulate musical exploration and knowledge from a unified yet open-ended environment: the IRiMaS software, which is the centre of attention of this paper. After a short discussion of existing software and their benefits and limitations for the study of music as sound, and of earlier software developments carried by the authors, the general design and current features of the IRiMaS software are presented. Finally, the contexts in which the software is used and evaluated – both currently and in forthcoming stages of the project – are discussed, along with the potential scope and impact of such an interactive aural approach[2] to general musicology.

2. Music Analysis and Existing Software

2.1 Investigation of music as sound in software

Assuming the use of a sound file as a starting point for the analysis of a musical work or performance, and regardless of whether that sound file is considered to be equivalent to the work itself (such as music for tape), a reference recording amongst possible others (such as the commercialized

[1] See the IRiMaS project website [1] for further information on the project itself.

[2] As presented in the next section, the concept of "interactive aural approach" was developed by Michael Clarke in the context of the analysis of electroacoustic music [3].

recording of one performance of a classical piece), or a trace documenting a particular musical event (such as a free improvisation or a performance of strictly oral folk music), several readily available software environments can be used to undertake the musicological investigation.

Developed at the Centre for Digital Music, Queen Mary University of London, Sonic Visualiser [4] is an "open source application for viewing, analysing, and annotating music audio files", and has been used in different domains of musicology[3]. Providing waveform and sonogram representations, Sonic Visualiser can host Vamp plugins for audio feature extraction, such as Constant-Q transform [5], pitch estimation for polyphonic music [6], or structural segmentation [7], to name but a few. Another sound-based software with a strong musicological orientation is Pierre Couprie's EAnalysis, developed in collaboration with De Montfort University [8]. On the basis of several generic representations of sound, like waveform and sonogram, it enables its users to create graphic and textual annotations on multiple views, and incorporates MIR-related representations such as similarity matrixes or charts based on Malt and Jourdan's BStD (Brilliance and spectral standard deviation) [9]. Other pieces of software that don't have any particular musicological focus, but implement advanced audio analysis and synthesis features for creative purposes, such as Ircam's Audiosculpt [10], or Michael Klingbeil's SPEAR [11], can be very helpful for scrutinising, manipulating, and resynthesizing the audio contents of a musical work or excerpt in a musicological context.

While all these software environments have specific orientations, features, and user interfaces[4], they tend to focus the user's attention onto one object – the investigated sound file – and its inherent properties; they do not facilitate the manipulation and arrangement of a large set of items of heterogeneous natures and formats. Yet, in a musicological situation, a typical need is that of confronting different types of information, of segmenting audio or other data flexibly, and of arranging representations according to layouts that do not necessarily correspond to the timeline of the initial sound source.

2.2 Previous developments at the University of Huddersfield

To an extent, some previous developments by the contributors of this article prefigure the IRiMaS software, its requirements, and its features. In particular, the concept of "interactive aural analysis" developed by Michael Clarke [3] promotes the making and dissemination of musicological analysis in the form of software in order to enable both the researcher and the reader to engage aurally and interactively with the music, its form, its components, and its tools. Within an earlier project, the UK's Arts and Humanities Research Council funded TaCEM (Technology and Creativity in Electroacoustic Music) project[5], a generic software was prototyped: T*IAA*LS (Tools for Interactive Aural Analysis) [13]. Inspired by some features of AudioSculpt, it enables its users to navigate the sonogram of the studied sound file by scrubbing the FFT data at any coordinates across the time and frequency axes, and to draw shapes on the sonogram which act as filters to listen to them, isolated from the rest of the spectrum. In regards to the environments mentioned in section 2.1, one of the most distinctive features of T*IAA*LS is that the objects drawn on the sonogram can then be placed on an interactive chart, on which representations of the time-frequency segmentations of the original file can be arranged in any order on the two-dimensional plane, according to the specifications of the analyst, such as genealogies, typologies, or paradigmatic representations. The sound objects can be played directly from the chart itself, facilitating direct aural comparisons between objects and user-defined categories[6].

The central development work of the TaCEM project, however, was the realization of nine standalone applications, each dedicated to the interactive aural analysis of case studies from the electroacoustic repertoire, including key works of computer music such as John Chowning's *Stria* (1977) [15] and Barry Truax's *Riverrun* (1986) [16]. These nine applications include interactive presentations enabling users to familiarize themselves with the technologies developed and used by the composers, which is not directly relevant to IRiMaS, but also interactive charts presenting the composer's audio materials, again with genealogies, paradigmatic, or chronologic arrangements. In all cases, the charts enable the playback of all the displayed sound files, but also, where relevant, the possibility to focus the listening on specific sound objects or events rather than the entire work. Besides, the implementation in software made the visualization features possibly dynamic, so that a given chart can be arranged differently according to specific criteria[7]. The TaCEM applications also include ethnographic materials in the form of recorded videos, with interviews, demonstrations, and studio discussions with the composers and their collaborators.

While all these applications are dedicated to specific musical works and were designed by the TaCEM team, many of their features can be generalized to the study of any work, much beyond electroacoustic music, and it is precisely such an abstraction that the IRiMaS software aims to provide to its users: the ability to develop their own fully featured interactive aural analyses, with a range of investigation and presentation tools for the manipulation of any relevant type of data (audio and derived features, video, text), with no programming skills required.

[3] For instance, in Nicholas Cook's article on Webern's Piano Variations mentioned earlier [2].
[4] For a more developed list of existing software and a discussion of their benefits and limitations in a musicological context, see [8].
[5] Conducted at the University of Huddersfield and Durham University by Michael Clarke, Peter Manning, and Frédéric Dufeu. See [12].
[6] The idea of the generic interactive chart of T*IAA*LS was itself a development of charts built for an interactive aural analysis of Denis Smalley's *Wind Chimes* developed by Michael Clarke, in which the user could play the chart items, but not edit the chart itself [14].
[7] For instance, in the case of *Stria*, the main interactive chart enables the user to visualize all the elements making the work ordered according to any of their synthesis parameters.

3. Design of the IRiMaS Software

Written in C++ using the JUCE framework, the IRiMaS software is being developed by the project team through the five years of the project (2017-2022) and, at the time of writing this article, its general design and first features are stabilized. In the current stage of the project, the software is being used within the team with specific musicological tasks and requirements, as part of doctoral research on three case studies in different musical areas, as presented more precisely in section 4. In later stages, the software will be disseminated widely, and must be general enough to serve a large range of musicological domains and needs.

3.1 Overall architecture: objects, pages, and presentation

The IRiMaS software is designed around three main concepts to be considered by the user: the objects, the pages, and the presentation.

3.1.1 Objects

Objects are the most elementary component that the user interacts with; they are classes with specific functions that can be used as tools for playback and listening, audio analysis, editing, and presentation. They can be simple, such as text boxes in which the user writes annotations, headings, or anything more developed; or they can be complex, such as a sonogram that the user can navigate both aurally and visually, and from which time-frequency regions can be identified and isolated. Objects can be created from any pre-existing type of media (sound, text, graphics, video)[8], but an important specificity of the IRiMaS software is that the user can fluidly generate objects from other objects.

For instance, once a waveform object has been created by selecting an audio file in the user's library of sounds, the audio data can simply be played, entirely or within a specific region; markers and labels can be placed on the visualization to facilitate the navigation during a first aural exploration. The audio can be segmented, either manually or with algorithmic assistance; any segment can then be turned into an object in its own right. In a musicological context, this is a typical requirement: to take a schematic example, if a musical sequence has a form A B A' B A", it is crucial for the researcher to be able to rearrange each of the formal segments on a space that allows for comparative listening of the instances of B or of the variations of A, without going through the successive steps that such a simple requirement would imply in a standard digital audio workstation or audio editor.

3.1.2 Pages

The manipulation of audio segments as objects with their own graphical user interface in the example above introduces the concept of page, which is governed by two major functions. The first is that of the investigation itself: by arranging objects and relating them to each other on a unique space, the user interacts with the page as her/his workspace. The creation and manipulation of audio and other types of objects, the derivation of segments from waveforms or sonograms, the use of audio descriptors to guide aural comparisons or to support classifications, all contribute to the investigation of music as sound. The other essential function of the page is that of the dissemination: it is in the first instance with the page that the researcher presents a specific argumentation to her/his audience, by playing specific musical excerpts, displaying relevant representations, arranging as many objects as needed onto a fully user-defined, interactive and, perhaps most importantly, aural chart. Inspired by graphical and object-based environments like Pure Data or Max, the IRiMaS software implements for the page and its constitutive objects an edit mode and a presentation mode. In the edit mode, the musicologist makes most of the investigation, creates, edits, and arranges objects on the page[9] (Figure 1); in the second mode, she/he presents them according to the desired layout, and interacts with reduced means with the page to play the media corresponding to the objects, or to perform other simple operations.

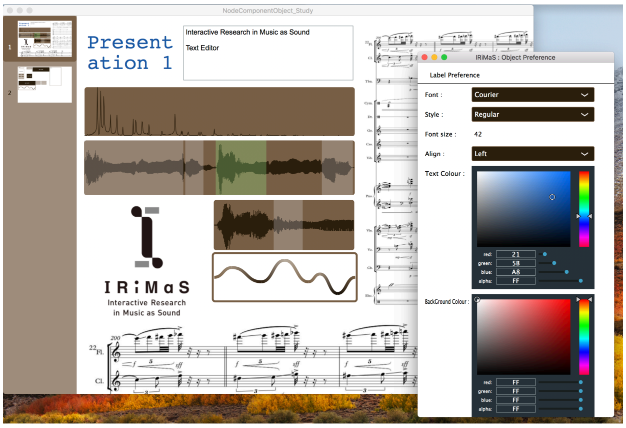

Figure 1. Example of page being edited in the IRiMaS software

3.1.3 The Presentation

The third main concept of the IRiMaS software is that of presentation: the user is free to create as many pages as necessary during the musicological investigation, and these pages ultimately make the presentation, which can be considered as an interactive research output. As with standard presentation software, the pages can be ordered as an indexed array and presented linearly; but, given the potential heterogeneity of resources often found in work of ethnographic nature, some alternative page navigators are

[8] A description of current and planned objects can be found in paragraph 3.3. below.

[9] The most fundamental operations are performed directly on the objects themselves, but for advanced operations or the definition of general attributes, objects are associated with an extrinsic inspector.

under study, such as trees, graphs, or maps[10]. More generally, pointers to specific pages can easily be created by the presentation designer (i.e. the musicologist) from any object on any page.

3.2 Typical workflow

To summarize the overall design of the IRiMaS software, the typical workflow of a musicologist using this integrated environment from the first steps of the investigation to the dissemination of the research outcomes themselves can be described as follows.

When launching the IRiMaS software, a new presentation is created with one blank page and no object on it. The user can create objects, either from scratch (text boxes, basic graphics) or, more typically, by importing audio or other media, such as static graphics or video. By default, importing an audio file creates a waveform object; it is also possible to create an interactive sonogram, either by importing audio and implicitly performing an FFT and generating its graphic representation, or from an existing waveform object. The user can in the first instance investigate the audio by playing the waveform or the sonogram, and segment it in time-only or time-frequency regions respectively. New objects can be created from such a segmentation; objects can be arranged on the page according to the musicologist's needs. Segmentation can be operated entirely manually; it can also be assisted computationally. With this example, as well as with the more advanced sound analysis algorithms to be described below, it is important to note that the IRiMaS software encourages the use of computational operations (in this example, segmentation) as a basis for human-based refinement, so that any set of segment markers proposed by an IRiMaS object can – and, in most cases, should – be edited by the user: in other words, the software proposes and assists, the musicologist decides. Arranging objects on the page can also be assisted computationally: for instance, segments of sound can be ordered according to average values of audio descriptors; self-organizing maps [18] can also help evaluating similarity and difference of a range of materials according to specific criteria, such as rhythmic motifs or melodic profiles. Once again, playing and listening being primary to the IRiMaS philosophy, playing audio objects individually, but also comparatively (by selecting several objects, playing them simultaneously, and soloing/muting them), is made immediately accessible in the environment.

Once the relevant objects have been created and arranged on the page, the researcher can refine the presentation by adding textual annotations and graphics; it is also possible to link objects together, for example so that clicking on a score image triggers the corresponding audio playback.

Finally, once an ensemble of pages has been made, the user can organise her/his presentation by selecting a specific presentation configuration (linear or map) and, where appropriate, implement inter-page links directly from the pages and objects themselves. A presentation is saved as an IRiMaS package (of the open-source .irimas format), which can be read and modified using the IRiMaS software.

3.3 Current and future objects and functions

As mentioned in the introduction of this article, the development of the IRiMaS software spans over the duration of the ERC project (2017-2022). At the time of writing this article, its first version already in use internally (see next section), and it is stable enough to demonstrate the general environment, its design and philosophy, and a first set of objects. The current list of objects is bound to expand as the project progresses, and as the software receives iterations of feedback by different audiences such as that of the International Computer Music Conference. As it stands, the following objects are in place and operational:

- Audio player with waveform representation, including time-domain segmentation, and several basic audio descriptors visualization;
- Audio player with sonogram representation, including the features of the audio player with waveform representations;
- Video player with specific controllers for use in a musicological context, facilitation of labelling and interaction with text;
- Static graphic file display, with temporal cursor for synchronisation with audio data;
- Text box for general annotation.

The following objects and functions are currently under development:

- Score editor with dynamic highlighting, for synchronisation with audio and Midi data;
- Assisted classification by audio features or musical parameters, including self-organizing maps;
- Partial tracking;
- Source separation of audio files;
- Playback visualisation de-solidarized from the conventional x or y axes conventional representations of time[11].

4. From the IRiMaS Case Studies to General Musicology

Three musicological case studies are built in the IRiMaS project; each of them focuses on a musical domain which has been traditionally challenging standard notation, and therefore important to consider for the project implementation, its interactive aural approach, and the development of the IRiMaS software. Each case study is led by a PhD

[10] The implementation of several modes of presentation and navigation from page to page aims to develop previous initiatives by two members of the IRiMaS project: Amanda Bayley and Michael Clarke, who published an interactive DVD gathering many different types of materials in an ethnographic study of Michael Finnissy's second String Quartet and giving the viewer many possible paths and navigation modes through these materials [17].

[11] After an idea of one of the team members of the IRiMaS project, Robert Adlington, who questions the validity of the metaphor of motion regarding musical experience [19].

student, whose analytical needs are addressed by the software development team.

An ethnomusicological case study is undertaken by Cristina Ghirardini and supervised by Michael Clarke and Jonathan Stock (University College Cork). This research focuses on improvised poetry in *ottava rima* in central Italy. The study of such a musical practice, essentially transmitted orally, is difficult to tackle on the basis of transcriptions by classical notation, which may convey to some extent pitches and durations, but fails to identify the numerous timbral variations or specific nuances in melodic profiles according to individual poets. The IRiMaS software aims not only at providing audio descriptions that are more relevant to the nuances of sound, such as those available in existing software as discussed in section 2, but more importantly at providing an environment in which segmenting stanzas and making aural comparisons of performances, iambic lines, and profiles of rhymes, can be made smoothly, as well as the scrutiny of the properties of sound and scansion.

A case study on free improvisation is led by Maria Donohue under the supervision of Michael Clarke and Amanda Bayley (Bath Spa University). Focused on interactions amongst collectives of improvisers, this research gives a particular attention to how the musicians observe each other (both aurally and visually), their responsiveness, and decision-making. Such elements require the definition of *ad hoc* modes of representation, which can be usefully supported by video materials documenting, for example, the point of view of each improviser. Here, the definition of relevant factors and descriptors representing ways by which the collective interactions contribute to the emergence of an ephemeral musical form challenges notation, and an interactive aural format can foster the observation and analysis of the music and its practice on the basis of specifically designed criteria, along with more standard descriptors for music and sound.

The repertoire of contemporary classical music is known for having led to a high individualisation of the musical languages and modes of notation; the third IRiMaS case study, undertaken by Laurens van der Wee and supervised by Michael Clarke and Robert Adlington (University of Huddersfield), focuses on spectralism. Unlike the two previous case studies, this repertoire does rely on notation, but what is written on the score often does little to reflect the aural manifestation of the music. Working on the basis of audio recordings, with playing and listening as primary activities as opposed to reading, is here again key to analysing music, its form, and its elementary components: the meaning of music is here conveyed by the composition of timbre, and the classical parameters of pitches, dynamic, and durations are insufficient material for the analyst. Since the representation of timbre has been a significant challenge for several fields of research, identifying, designating, and organising audio segments in a readily aural and interactive environment is likely to enable the musicologists of this area to investigate and present their findings directly in sound, the first model and metaphor for such compositions.

With these three case studies, tackling three very different musical domains, all challenging for musical notation and therefore calling for new analytical methodologies, the IRiMaS software addresses specific musicological requests, and the development of the software in turn provokes musicological and analytical ideas which would otherwise not emerge. But as its implementation progresses, the software is being considered for a wider field of musics: not only those for which notation is problematic, but also the ones already tackled by score-based musicology. Implementing interactive aural versions of existing analyses is targeted by the IRiMaS project[12]; the implementation of models in which the user of the software can play with notation and motif combinations to literally experiment in the same way the composer could may well foster new modes of musicological knowledge for notation-based repertoire, provided that the aural experience is made accessible integrally.

5. Conclusion

This article has introduced the IRiMaS software, currently developed as part of the ERC-funded project directed by Michael Clarke. The concept of interactive aural analysis, previously designed for the field of electroacoustic music and computer music, is key to the development of an emerging mode of practicing musicology: that in which the analysis focuses on the aural, letting both the researcher and her/his audience play with and listen to the music, its form, its components, and its processes. To be pushed beyond a simple hypermedia stage as in some recent literature, such an approach requires significant software development, in order to facilitate the identification and definition of relevant musical elements, their comparison, their organisation, and their presentation. Such requirements can largely be facilitated by advanced digital signal processing techniques found in existing platforms, but no existing software fully integrates the aural in a unified environment that considers, along with low-level sound description, musicological needs like the comparison, ordering, and arrangement of multiple elements. Providing advanced tools for the design of user-defined interactive aural analyses must enable musicologists with no programming skills to elaborate and disseminate their investigations with sound as a primary medium. It is with the full integration of musicological knowledge and aural experience as first principle that the IRiMaS software is designed, and it is hoped that the presentation of its successive iterations through the five years of the project will benefit from feedback from both the musicological and the signal analysis communities.

[12] For instance, Jean-Jacques Nattiez's analysis of Varèse's *Density 21.5* [20].

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